

On the absence of winds in ADAFs

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ABSTRACT

We show that recently published assertions that advection dominated accretion flows (ADAFs) require the presence of strong winds are unfounded because they assume that low radiative efficiency in flows accreting at low rates onto black holes implies vanishing radial energy and angular momentum fluxes through the flow (which is also formulated in terms of the ‘Bernoulli function’ being positive). This, however, is a property only of self-similar solutions which are an inadequate representation of global accretion flows. We recall general properties of accretion flows onto black holes and show that such, necessarily transonic, flows may have either positive or negative Bernoulli function depending on the flow viscosity. Flows with low viscosities ($\alpha \lesssim 0.1$ in the α -viscosity model) have a negative Bernoulli function. Without exception, all 2-D and 1-D numerical models of low viscosity flows constructed to date experience no significant outflows. At high viscosities the presence of outflows depends on the assumed viscosity, the equation of state and on the outer boundary condition. The positive sign of the Bernoulli function invoked in this context is irrelevant to the presence of outflows. As an illustration, we recall 2-D numerical models with moderate viscosity that have positive values of the Bernoulli function and experience no outflows. ADAFs, therefore, do not differ from this point of view from thin Keplerian discs: they may have, but they do not have to have strong winds.

Key words:

accretion: accretion discs — black holes physics — hydrodynamics

1 INTRODUCTION

In many systems containing accreting galactic and extragalactic black holes, the luminosity deduced from observations is much lower than the one obtained by assuming a ‘standard’ radiation efficiency of ~ 0.1 . It has been proposed that accretion in such systems can be modeled by advectively dominated accretion flows – ADAFs (for recent reviews see Kato, Fukue & Mineshige 1998; Abramowicz, Björnsson & Pringle 1998; Lasota 1999). In ADAFs, the main cooling process is advection of heat — radiative cooling is only a small perturbation in the energy balance and has no dynamical importance. ADAFs are quite successful

in explaining observed spectral properties of accretion onto black hole in low mass X-ray binaries, Galactic center, and some active galactic nuclei.

Recently, Blandford & Begelman (1999, hereafter BB99) put in question the physical self-consistency of ADAF models by arguing that flows with small radiative efficiency should experience outflows, which in some cases could be so strong as to prevent accretion of matter onto the black hole. Low luminosities, therefore, would not be due to low radiative efficiency, but simply to absence of accreting matter near the central object. Hence, ADAFs should be replaced by what BB99 call ADIOs, i.e. ‘advection-dominated inflows-outflows’ (later ‘advective’ has been changed to ‘adiabatic’). The BB99 argument about the necessity of outflows from ADAFs is based on two statements about what they call the ‘Bernoulli constant’ (the significance of this ‘constant’ - in reality a function - is discussed in the next Section): [1] The Bernoulli constant in flows with low radiative efficiency must always be positive, a point which was first

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made by Narayan & Yi (1994), [2] A positive Bernoulli constant implies outflows.

In the present paper we show that [1] the vertically integrated Bernoulli function is *everywhere negative* in small viscosity ADAFs which have a vanishing viscous torque at the flow inner ‘edge’, and are matched to the standard thin Shakura-Sunyaev disc (SSD) at the outer edge, [2] A positive Bernoulli function in an ADAF does not imply outflows — in a representative class of 2-D numerical models of ADAFs with moderate viscosity, Bernoulli function may be positive but despite of that outflows are always absent.

The main conclusion of our paper is that while it is not yet clear whether some ADAFs with high viscosity could indeed have significant outflows, both general theoretical arguments and numerical simulations point out that it is unlikely that ADAFs with low viscosity could experience even moderate outflows of *purely hydrodynamical* origin. Similar conclusion has been recently obtained by Nakamura (1998), who used a different approach.

We use here cylindrical coordinates (r, z, ϕ) and denote gravitational radius of the accreting black hole (with the mass M) by $r_G = 2GM/c^2$. We model the gravitational field of the black hole by the Paczyński & Wiita (1980) potential. We assume in this paper that viscosity is small. This means that $\alpha \lesssim 0.1$ for the kinematic viscosity coefficient given by a phenomenological formula

$$\nu = \alpha \ell_P C_{\text{sound}}, \quad (1.1)$$

where $\ell_P = P/|\nabla P|$ is the pressure length scale and C_{sound} is the sound speed. Our theoretical arguments do not depend on a particular functional form (prescription) of α . In numerical models we assume the ‘standard’ prescription, $\alpha = \text{const}$. Recent theoretical estimates based on numerical simulations of turbulent viscosity (see e.g. Balbus, Hawley & Stone 1996) show that most likely $\alpha \lesssim 0.1$. In this paper we shall consider mainly this *low viscosity* range of α . Arguments based on fitting predicted spectral properties of ADAF models to observations (see Narayan 1999 for review) seem to require the *moderate viscosity* range, $0.1 \lesssim \alpha \lesssim 0.3$, that partially overlaps with the low viscosity range considered in this paper. As pointed out by Rees (private communication) it is not clear if the *high viscosity* range, $\alpha \gtrsim 0.3$, is physically realistic.

2 THE BERNOULLI CONSTANT, FUNCTION AND PARAMETER

In stationary, inviscid flows with no energy sources or losses, the quantity,

$$B_0 = W + \frac{1}{2}V^2 + \Phi = \text{const}, \quad (2.1)$$

is constant along each individual stream line, but, in general, is different for different stream lines. This quantity is called the Bernoulli constant. Here W is the specific enthalpy, V is the velocity (all three components included), and Φ is the gravitational potential. Obviously, a particular streamline may end up at infinity *only if* $B_0 > 0$ along it. The existence of stream lines with $B_0 > 0$ is therefore a *necessary* condition for outflows in stationary inviscid flows with no energy sources or losses, and $B_0 < 0$ for all streamlines is a *sufficient* condition for the absence of outflows. However,

$B_0 > 0$ is *not* a sufficient condition for outflows. For example, in the case of the classical Bondi’s accretion (pure inflow) B_0 is a universal positive constant.

In all viscous flows, B_0 defined by (2.1) is, of course, *not* constant along individual stream lines. However, the so-called ‘Bernoulli parameter’,

$$\tilde{B}_0 = \frac{1}{V_K^2} \left[W + \frac{1}{2}V^2 + \Phi \right], \quad (2.2)$$

is a universal constant in 1-D, vertically integrated, self-similar Newtonian models of ADAFs introduced by Narayan & Yi (1994). Here V_K is the Keplerian velocity. The reason for that is simple: in self-similar models all quantities scale as some power of the cylindrical coordinate r . In particular, because both

$$B = W + \frac{1}{2}V^2 + \Phi \neq \text{const}, \quad (2.3)$$

and V_K^2 scale as r^{-1} , their ratio \tilde{B}_0 must be a constant. Narayan & Yi (1994) and BB99 have argued that because the vertically integrated models have $\tilde{B}_0 > 0$, in generic 2-D flows there should be streamlines escaping to infinity.

This argument is not correct for the simple reason that neither $B_0 > 0$ (as already mentioned), nor $\tilde{B}_0 > 0$, are sufficient conditions for outflow existence. Note that $\tilde{B}_0 = \text{constant}$ is not a real physical conservation law, but rather an artifact induced by a purely mathematical assumption that the flow is self-similar. This *very strong* assumption is motivated only by practical convenience, not by the physical properties of a flow. Indeed, in all numerical, global models of ADAFs constructed so far, self-similarity does not represent well the global flow properties and \tilde{B}_0 changes its value and even its sign from place to place. Large scale changes are due to the global balance between viscous heating, advective cooling and PdV work. Small scale changes are typical for ADAFs in which a strong convection (circulation) is present (these ADAFs have a moderate or small viscosity). In this case, the sign of \tilde{B}_0 changes rather abruptly, tracing convective bubbles. In addition, at a given place, the value and the sign of \tilde{B}_0 strongly fluctuates in time.

The Bernoulli constant B_0 is not a useful quantity for ADAF study, because they are viscous flows. The Bernoulli parameter \tilde{B}_0 has no physical significance and therefore it is not a convenient quantity for theoretical arguments. We shall use here the ‘Bernoulli function’, which is formally defined by (2.3), but — of course — in ADAFs (or any other viscous flows) it is *not* constant along streamlines. As in the case of steady dissipation-free flows, $B < 0$ everywhere is a sufficient condition for the absence of outflows.

We were forced to introduce such new object 217 years after the death of Daniel Bernoulli because of the above-mentioned articles on outflows from ADAFs, which use the related concepts of the Bernoulli constant or ‘parameter’. It is easier, as shown below, to refute their arguments by using the same ‘paradigm’.

3 THE INNER BOUNDARY CONDITION

In this Section we would like to stress the importance of the fact that the accreting body is a black hole, in particular, the implications of the transonic nature of the accretion flow.

We will assume that the flow viscosity is low. Paczyński

(1978, unpublished) noticed that in this case the mass loss from the inner part of an accretion disc around a black hole is fully governed by the general relativistic effect of ‘relativistic Roche lobe overflow’ which operates close to a ‘cusp’ radius r_{cusp} , where the angular momentum of the disc takes the Keplerian value, $j(r_{cusp}) = j_K(r_{cusp})$ (see Abramowicz 1981, 1985 for details). Here $j_K = V_K r$ is the Keplerian angular momentum. The cusp is formed by a critical equipotential surface, similar (but topologically different) to the critical Roche surface in the binary stellar-system problem.

Jaroszyński, Abramowicz & Paczyński (1980) proved that for small viscosity accretion discs one must have $r_{mb} < r_{cusp} < r_{ms}$, where r_{mb} and r_{ms} are respectively the radii of the marginally bound, and the marginally stable Keplerian circular orbits around a black hole. In the case of a non-rotating (Schwarzschild) black hole, as well as in the Paczyński & Wiita (1980) potential, $r_{mb} = 2r_G$ and $r_{ms} = 3r_G$. At r_{ms} the Keplerian angular momentum has a minimum value $j_{ms} = (3/2)^{3/2} r_G c$. Numerical simulations show that low viscosity accretion discs with reasonable *outer* boundary conditions (discussed in the next Section) have always a *high specific angular momentum*, that is for $r_{cusp} < r < r_0$, they are always super-Keplerian: $j(r) > j_K(r)$, where $r_0 \approx 5r_G$ (see Figure 1).

The mass loss through the cusp, at the rate \dot{M}_{cusp} , induces an advective cooling Q_{cusp}^- which is a very sensitive function of the vertical thickness h of the disc at the cusp: independent of the equation of state (i.e. independent of the adiabatic index of the accreted matter γ) one has $Q_{cusp}^- \sim \Sigma h^3$, where Σ is the disc surface density at the cusp. This implies that the relativistic Roche lobe overflow stabilizes possible unstable thermal modes in the region close to r_{cusp} because an overheating would cause vertical expansion, which then would induce strong advective cooling. More precisely, in terms of Pringle’s thermal stability criterion (Pringle 1976; Piran 1978),

$$\left(\frac{\partial \ln Q_{vis}^+}{\partial \ln h} \right)_\Sigma < \left(\frac{\partial \ln Q_{cusp}^-}{\partial \ln h} \right)_\Sigma, \quad (3.1)$$

where $Q_{vis}^+ \sim \Sigma h^2$ is the rate of viscous heating, one has $2 = (l.h.s) < (r.h.s) = 3$, which proves thermal stability (Abramowicz, 1981). Thus, an increase in the Bernoulli function caused by overheating does not produce outflows, but enhances the inflow into the black hole. This analytic prediction has been confirmed in all details by 2-D time dependent numerical simulations. In particular, Igumenshchev, Chen & Abramowicz (1996) showed that the analytic formula (Abramowicz, 1985) for the rate of the mass inflow into the black hole induced by the relativistic Roche lobe overflow reproduces, in a wide range of parameters and with impressive *quantitative* accuracy, the behavior of \dot{M}_{cusp} calculated in *all* their numerical simulations.

For low viscosity (high angular momentum) black hole accretion flows, Abramowicz & Zurek (1981) found that the regularity condition at the sonic point $V_r^2 = C_{sound}^2$ requires

$$\left(\frac{C_{sound}}{c} \right)^2 \equiv \varepsilon^2 = \frac{j_K^2(r_{sonic}) - j^2(r_{sonic})}{2r_{sonic}^2 c^2}. \quad (3.2)$$

Because $\varepsilon \ll 1$ and $j_K(r_{sonic}) \approx r_{sonic} c$, one has

$$\frac{j(r_{sonic})}{j_K(r_{sonic})} = 1 - \frac{r_{sonic}^2 c^2}{j_K^2(r_{sonic})} \varepsilon^2 + \mathcal{O}(\varepsilon^4), \quad (3.3)$$

and

$$\frac{r_{sonic}}{r_{cusp}} = 1 - \mathcal{O}(\varepsilon^2), \quad (3.4)$$

i.e. the sonic point is very close to the cusp. These properties have been confirmed by 1-D and 2-D numerical simulations performed independently by numerous authors.

The supersonic flow at $r < r_{sonic}$ does not hydrodynamically influence the subsonic flow at $r > r_{sonic}$. For this reason, somewhere at an ‘inner edge’ $r_{in} \approx r_{sonic} \approx r_{cusp}$, the viscous torque vanishes, $g(r_{in}) = 0$. From (2.3) and (3.2) – (3.4) one derives, for a polytropic fluid, i.e. with $W = C_{sound}^2/(\gamma - 1)$,

$$\frac{B(x_{sonic})}{c^2} = -\frac{x_{sonic} - 2}{4(x_{sonic} - 1)^2} + \frac{3 - \gamma}{2(\gamma - 1)} \varepsilon^2 + \mathcal{O}(\varepsilon^4). \quad (3.5)$$

Here $x \equiv r/r_G$. In the physically relevant region, $2 \leq x_{sonic} \leq 3$, the leading term (zeroth order in ε^2) of this function equals, independently of γ , to *negative* binding energy at circular Keplerian orbit and varies between 0 and $-1/16$.

Thus, for ADAFs with low viscosity (high angular momentum) one *should* adopt the following inner boundary condition,

$$g(r_{in}) = 0, \quad j(r_{in}) = j_K(r_{in}). \quad (3.6)$$

because this is imposed by fundamental properties of the black hole gravitational field and thus *must be* always obeyed. In addition, if at the sonic point $C_{sound}/c \ll 1$, the sonic point regularity condition demands that the Bernoulli function at the inner edge should be negative,

$$B(r_{in}) \approx B(r_{sonic}) < 0, \quad (3.7)$$

independent of the equation of state (independent of γ).

4 THE OUTER BOUNDARY CONDITION

ADAFs cannot exist for arbitrary large radii. For example, according to Abramowicz et al. (1995) ADAFs cannot extend beyond the radius

$$r_{max} = C \frac{\alpha^4}{m^2} r_G \gg r_{in}, \quad (4.1)$$

where $\dot{m} = \dot{M}/\dot{M}_{Edd}$ and $C \simeq 10^2$. In general, C and the power of α depend on the cooling mechanisms included into the model (see Menou, Narayan & Lasota 1999).

Observations suggest that, in several systems the inner ADAF is surrounded by a geometrically thin, standard Shakura-Sunyaev disc so that there must exist a transition region where, for $r \sim r_{out}$ the ADAF is matched to a Keplerian disc. The physical mechanism which triggers the transition is not known (see, however, Meyer & Meyer-Hofmeister 1994; Kato & Nakamura 1998) and it is not clear what is the relation between r_{max} and r_{out} .

These uncertainties make the conditions at the outer edge r_{out} of low viscosity accretion discs less precisely determined than the conditions at the inner edge. Despite of this, Abramowicz, Igumenshchev & Lasota (1998) found a simple analytic argument that proves that independent of the physical reason for the transition, the angular momentum close at r_{out} should have exactly the Keplerian value, $j(r_{out}) = j_K(r_{out})$. This was confirmed by numerical models

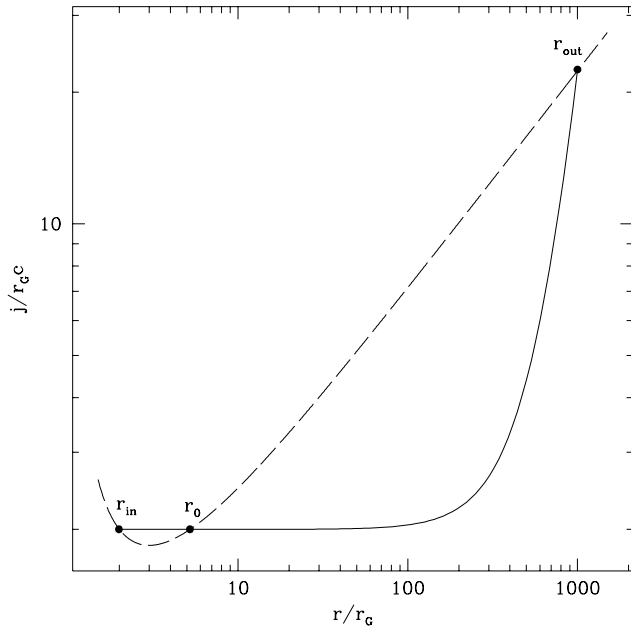


Figure 1. Angular momentum distribution for low viscosity ADFAF (solid line) that fulfills inner (3.6) and outer (4.2) boundary conditions, given by Paczyński's fitting formula (5.2). The dashed line corresponds to the Keplerian distribution $j_K(r)$.

of the only specific ADFAF-SSD transition model worked out to date, in which the transition occurs due to the presence of a turbulent flux (Honma 1996, Kato & Nakamura 1998).

With an ADFAF joining the SSD at the outer edge, one has,

$$j(r_{out}) = j_K(r_{out}), \quad B(r_{out}) \approx e_K(r_{out}) < 0, \quad (4.2)$$

Here $e_K(r_{out})$ is the *negative* Keplerian orbital binding energy. The second equation in (4.2) follows from

$$B(r_{out}) = e_K(r_{out}) [1 + \mathcal{O}((h_{out}/r_{out})^2)] < 0, \quad (4.3)$$

where $h_{out} \ll r_{out}$ is the thickness of the outer SSD. In all numerical simulations of low viscosity, high angular momentum accretion flows, the outer boundary condition (4.2) is *approximately* fulfilled at all radii $r \gg r_{in}$ independent of whether the ADFAF-SSD transition occurs.

5 THE ANGULAR MOMENTUM

The shape of angular momentum distribution between r_{in} and r_{out} depends mainly on viscosity but, as numerous 1-D and 2-D models demonstrate, in the low viscosity case it is always similar to that shown in Figure 1. The physical reason for such a shape is clear. Close to the inner edge, the viscous torque is ineffective (note that $g(r_{in}) = 0$) and thus the specific angular momentum has a very small gradient. Thus, the location of r_{in} roughly determines the location of the first crossing point r_0 between $j(r)$ and $j_K(r)$ curves, and therefore also the $1/r^3$ weighted area A_{in} between these curves in the region $[r_{in}, r_0]$. Mechanical equilibrium condition demands that (Abramowicz, Calvani & Nobili 1980),

$$A_{in} = A_{out}, \quad (5.1)$$

$$A_{in} \equiv \int_{r_{in}}^{r_0} \frac{j^2(r) - j_K^2(r)}{r^3} dr$$

$$A_{out} \equiv \int_{r_0}^{r_{out}} \frac{j_K^2(r) - j^2(r)}{r^3} dr,$$

and thus the $1/r^3$ weighted area A_{out} between these curves in the region $[r_0, r_{out}]$ is also roughly determined. In addition, one should have $dj/dr > 0$ and $d(j/r^2)/dr < 0$.

The function $j(r)$ can be approximated by the analytic fitting formula used in the Paczyński's (1998) toy model for ADFAFs,

$$j(r) = j_{r_{in}} \left[1 + b \left(\frac{r}{r_{in}} - 1 \right)^a + b \left(\frac{r}{r_{in}} - 1 \right)^{3a} \right]^{1.5/a}. \quad (5.2)$$

Note that this formula is different from the one in the published version of Paczyński's article. The error it contained has been corrected in the astro-ph version. Here a and b are constants that depend on r_{in} and r_{out} , and can be determined from (5.1). Although in actual models the range of radii for which the angular momentum is approximately constant is much reduced, one should stress, that (5.2) is a good qualitative representation of the *generic* angular momentum distribution in ADFAFs when the magnitude of viscosity is low *independent* of the functional form of viscosity.

6 THE BERNOULLI FUNCTION

Imagine a cylinder $r = \text{const}$ crossing an ADFAF from its upper, $z = h(r)$, to lower $z = -h(r)$, surface. Let \dot{M}_0 , \dot{J}_0 and \dot{E}_0 denote, respectively, the total amount of mass ($\dot{M}_0 < 0$), angular momentum and energy that cross the surface of the cylinder per unit time. In a steady state, from the Navier-Stokes equations of mass, angular momentum and energy conservations integrated along the cylinder it follows that,

$$\dot{M}_0 = \int_{-h(r)}^{+h(r)} 2\pi r \rho(r, z) V_r(r, z) dz = \text{const}, \quad (6.1a)$$

$$\dot{J}_0 = \dot{M}_0 j(r) + g(r) = \text{const}, \quad (6.1b)$$

$$\dot{E}_0 = \dot{M}_0 B(r) + \Omega(r) g(r) = \text{const}. \quad (6.1c)$$

Here $\Omega(r) = j(r)/r^2$ is the angular velocity. The radiative energy flux from ADFAFs is very small and it was ignored in (6.1c). Derivation of (6.1) assumes that the flow has an azimuthal symmetry (no dependence on ϕ), and that the orbital velocity V_ϕ is much greater than the accretion velocity V_r (and 'vertical' velocities) which is true, except very close to the inner edge, for flows with a small viscosity considered in this Section. Except $h(r)$, each radial function that appear in (6.1) represents the averaged value of the corresponding quantity. In particular,

$$B(r) = \frac{1}{2h(r)\dot{M}_0} \int_{-h(r)}^{+h(r)} 2\pi r \rho(r, z) V_r(r, z) B(r, z) dz. \quad (6.2)$$

The same equations (6.1), with the same assumptions and with the same averaging procedure (6.2), have been used by BB99, and by Paczyński (1998) in his recent illuminating paper on a toy model of ADFAFs.

According to Narayan and Yi (1994) in self-similar models of ADAFs, both $j(r)$ and $g(r)$ scale as $r^{1/2}$, and both $B(r)$ and $\Omega(r)g(r)$ scale as r^{-1} . These scaling properties imply that $\dot{J} = C_J r^{1/2}$ and $\dot{E} = C_E r^{-1}$, with $C_J = \text{const}$, $C_E = \text{const}$. Thus, the fluxes \dot{J}_0 and \dot{E}_0 can be constant if and only if $C_J = C_E = 0$. This implies that both the angular momentum flux, and the energy flux are exactly zero in self-similar models: $\dot{J}_0 = 0$, and $\dot{E}_0 = 0$ (BB99). From the first of these equations it follows that $g(r) = -j(r)/\dot{M}_0$. This, together with the second equation show that the Bernoulli function must be positive,

$$B(r) = \Omega(r)j(r) = [r\Omega(r)]^2 > 0. \quad (6.3)$$

The conclusion $B(r) > 0$ is based on self-similar solutions which are obviously inconsistent with boundary conditions. We shall show that by taking properly into account the boundary conditions (3.6) and (4.2), one arrives at the opposite conclusion: $B(r) < 0$.

The inner boundary condition $g(r_{in}) = 0$ implies that

$$\dot{M}_0 j(r_{in}) = \dot{M}_0 j(r) + g(r), \quad (6.4a)$$

$$\dot{M}_0 B(r_{in}) = \dot{M}_0 B(r) + \Omega(r)g(r). \quad (6.4b)$$

From the last two equations one derives

$$B(r) = B(r_{in}) + \Omega(r)j(r) - \Omega(r)j(r_{in}). \quad (6.5)$$

Equation (6.5) yields, at the outer edge of the disc r_{out}

$$B(r_{in}) = B(r_{out}) - \Omega(r_{out})[j(r_{out}) - j(r_{in})]. \quad (6.6)$$

Because in stable discs $j(r_{out}) > j(r_{in})$, the last term on the right hand side of this equation is always negative. From this fact and from equation (4.3) one concludes that for standard ADAFs, i.e. for those that have a vanishing torque at the inner edge r_{in} and match the standard Shakura-Sunyaev disc at the outer edge r_{out} one must have

$$B(r_{in}) < B(r_{out}) < 0. \quad (6.7)$$

Thus, the Bernoulli function in standard ADAFs must be negative both at the inner and outer edges. Identical conclusions have been reached, but not explicitly stated, by Paczyński (1998): see his equation (20) which is equivalent to our equation (6.6).

From equation (5.2) describing a typical angular momentum distribution in the disc, and equations (6.5), (6.6) one may calculate $B(r)$ in the whole disc. Figure 2 shows the Bernoulli function by the thick solid line. The thin broken line shows the prediction of the self-similar model. In the same Figure 2 we present for comparison by the thin solid line the time-averaged Bernoulli function for 2-D numerical model of ADAF with $\alpha = 0.01$. Details of numerical techniques described by Igumenshchev & Abramowicz (1999). The model has $r_{in} = 3r_G$, $r_{out} = 8000r_G$, and $\gamma = 5/3$.

One concludes that in ADAFs with small viscosity, which fulfill standard boundary conditions, the Bernoulli function must be everywhere negative. This conclusion is independent of the functional form of viscosity. Obviously, a flow which has the Bernoulli function that is everywhere negative does not experience outflows. Note, however that $B(r)$ calculated here is averaged with respect to the flux of mass. Thus, it may happen that $B > 0$ in some polar directions. This is indeed the case close to the disc surface of some low viscosity numerical models calculated by ICA96. These models show weak outflows, with $\dot{M}_{out} \ll \dot{M}_0$.

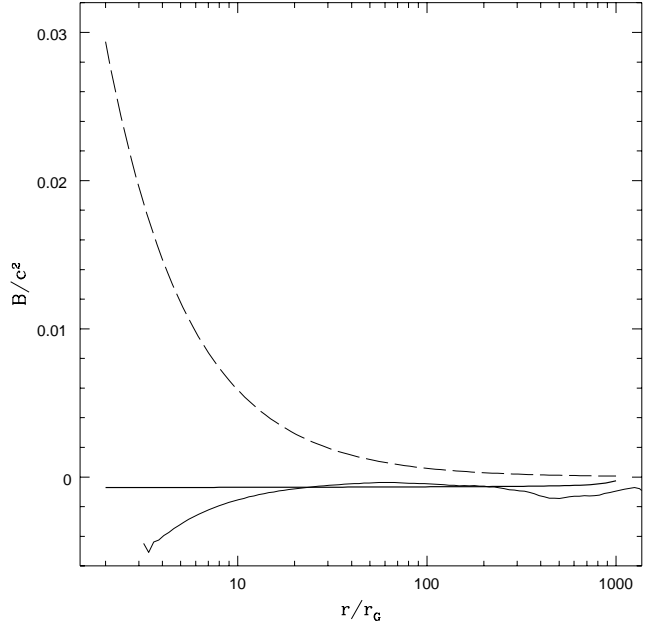


Figure 2. The Bernoulli function in ADAFs with small viscosity is everywhere negative. The solid thick line corresponds to the angular momentum distribution given in Figure 1, with $r_{in} = 2.003r_G$, $r_{out} = 10^3 r_G$. The thin dashed line corresponds to the Narayan & Yi (1994) self-similar solution (with $\gamma = 3/2$), not compatible with the boundary conditions, used by BB99. The thin solid line represents the time-averaged Bernoulli function for 2-D numerical model of ADAF with $\alpha = 0.01$.

7 NUMERICAL SIMULATIONS

Analytic arguments presented in the previous Section and pointing out that no significant outflows of hydrodynamical origin should be present in low viscosity ADAFs have been fully confirmed by all numerical simulations performed to date. Below we give a list of some representative works.

1-D global simulations of transonic flows in optically thick case (slim discs): Abramowicz, Czerny, Lasota & Szuszkiewicz (1988); Kato, Honma & Matsumoto (1988ab); Chen & Taam (1993); Szuszkiewicz & Miller (1997).

1-D global simulations of transonic flows in the optically thin case (ADAFs): Honma (1996); Chen, Abramowicz & Lasota (1997); Narayan, Kato & Honma (1997); Gammie & Popham (1998); Nakamura, Kusunose, Matsumoto & Kato (1998); Igumenshchev, Abramowicz & Novikov (1998); Ogilvie (1999).

2-D time dependent simulations of ADAFs: Igumenshchev, Chen & Abramowicz (1996); Igumenshchev & Abramowicz (1999); Stone, Pringle & Begelman (1999).

8 ADAFS WITH LARGE VISCOSITY

We have seen that there is a significant theoretical and numerical evidence which shows that purely hydrodynamical outflows in ADAFs with low viscosity are unlikely to occur. Physical reasons for the absence of outflows in this case seem to be well understood. They depend on some fundamental properties of the black hole gravity.

The situation in the case of moderate and large viscosity is less clear. The analytic calculation of $B(r)$ presented in

the Section 5 cannot be repeated in the case of large viscosity because in this calculation one assumes that $V_r \ll V_\phi$, while in flows with large viscosity, all velocity components are of the same order. This brings an additional unknown term to the energy equation (6.1c) so that the system has too many unknowns to be solved. Also, one can not further assume zero viscous torque acting at r_{in} , if non-circular motions are significant. Thus, equations (6.1) are insufficient to calculate $B(r)$ even in the self-similar case. In the large viscosity case one can argue neither that self-similarity implies $B(r) > 0$, nor that the boundary conditions imply $B(r) < 0$.

Purely hydrodynamic outflows have been seen in numerical 2-D simulations of high viscous accretion flows (Igumenshchev & Abramowicz 1999), and in 2-D self-similar models (Xu & Chen 1997, and references therein), but they are not a universal property of viscous flows. From the existing results it is obvious that the presence of outflows depends on the magnitude of viscosity parameter α , adiabatic index γ , and, probably, on the outer boundary conditions at r_{out} . However, the boundary $\mathcal{F}(\alpha, \gamma, \dots)$ that divide the parameter space into the flows with and without outflows is yet to be found.

Certainly, one cannot argue that $B > 0$ implies outflows. We illustrate this point by showing in Figure 3 a model of ADAF ($\alpha = 0.3$, $\gamma = 3/2$, $r_{in} = 3r_G$, $r_{out} = 8000r_G$) in which $B > 0$ and no outflows. The model has been calculated using numerical technics described by Igumenshchev & Abramowicz (1999). In Figure 4, for the same model, we show calculated distributions of $B(r)$ (solid line),

$$B(r) = \frac{2\pi r^2}{\dot{M}_0 c^2} \int_0^\pi \rho V_r \left(\frac{1}{2} V^2 + W - \frac{GM}{r} \right) \cos \theta d\theta, \quad (8.1)$$

normalized viscous energy flux (dashed line),

$$G(r) = -\frac{2\pi r^2}{\dot{M}_0 c^2} \int_0^\pi (V_r \Pi_{rr} + V_\theta \Pi_{r\theta} + V_\phi \Pi_{r\phi}) \cos \theta d\theta, \quad (8.2)$$

and normalized total energy flux (solid thick line),

$$\frac{\dot{E}_0(r)}{\dot{M}_0 c^2} = B(r) + G(r). \quad (8.3)$$

In (8.1)-(8.3) r , θ and ϕ are spherical coordinates and Π is the shear stress tensor. In the model the inward energy advection flux [term B in (8.3) and short-dashed line in Fig. 4] almost equals to the outward viscous energy flux [term G in (8.3) and long-dashed line in Fig. 4] at each radius. This behaviour is similar to that in the self-similar Narayan & Yi (1994) solution, where the oppositely directed fluxes exactly compensate. For comparison, we show $B(r)$ for the self-similar solution in Figure 4 by dotted line. When correct boundary conditions are taken into account there is no exact compensation and the total energy flux \dot{E}_0 must be a small (in absolute value) non-positive constant in the stationary dissipative accretion flow of the type discussed here. Note that in the actual model \dot{E}_0 (solid line in Fig. 4) is not constant and oscillates with a small amplitude due to an inaccuracy of our numerical scheme, which does not exactly conserve the total energy balance. This inaccuracy is inside $\approx 5\%$ of relative error, which is acceptable for our purposes.

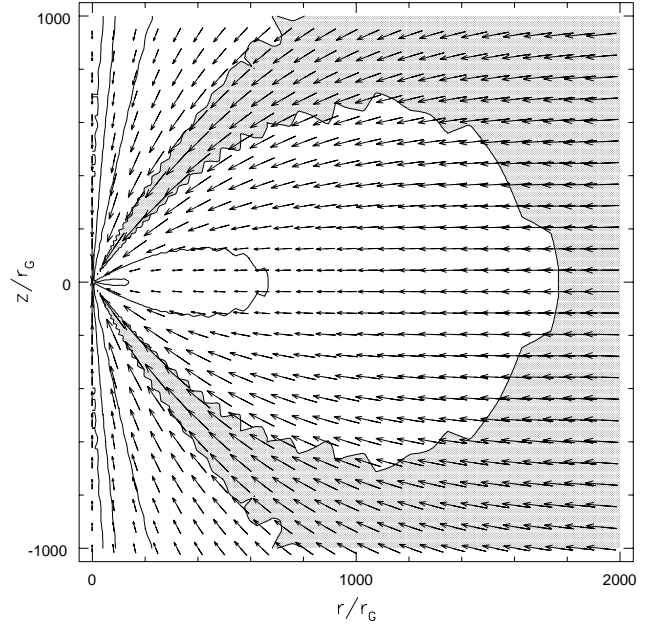


Figure 3. 2-D numerical stationary ADAF model with $\alpha = 0.3$, $\gamma = 3/2$, $r_{in} = 3r_G$ and $r_{out} = 8000r_G$. Only inner part of the model is shown. Arrows indicate the magnitude of the vector $r^2 \rho \vec{v}$, where r is the spherical radius. Regions with positive Bernoulli function are in white, while regions with negative values are in grey. Clearly, $B > 0$ is not sufficient for outflows.

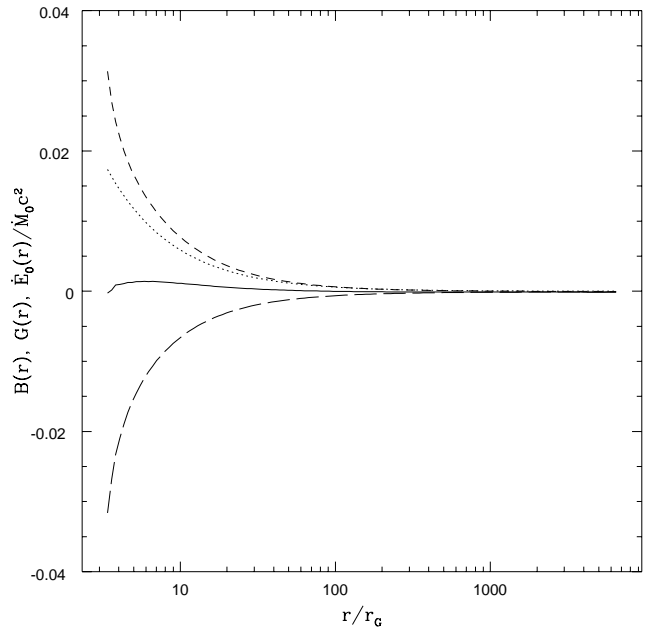


Figure 4. Radial profiles of θ -averaged quantities, given by eqs (8.1)-(8.3), for the model presented in Figure 3. Profiles of $B(r)$ (short-dashed line), $G(r)$ (long-dashed line) and $\dot{E}_0(r)/\dot{M}_0 c^2$ (solid line) are shown. The dotted line corresponds to $B(r)$ for the self-similar Narayan & Yi (1994) solution. See text for explanations.

9 CONCLUSIONS

1. Significant outflows of purely hydrodynamical origin are not present in ADAFs with low ($\alpha \lesssim 0.1$) viscosity. This conclusion follows from general theoretical arguments that involve fundamental properties of black hole gravity, and are well understood. All numerical simulations confirm this theoretical arguments and show no outflows.
2. It would be very interesting to perform a systematic investigation in the parameter space $\{\alpha, \gamma, r_{\text{out}}\}$ by filling it with models of ADAFs and find the regions (necessarily with $\alpha > 0.1$) corresponding to outflows.
3. Outflows from ADAFs might occur due to non-hydrodynamical factors such as magnetic fields, radiation, etc. These processes cannot be modeled in purely hydrodynamical terms by adopting special value of γ or form of the Bernoulli function: the extra physics should enter through solutions of the relevant equations (see e.g. King & Begelman 1999).

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